

1.

$$\begin{array}{r}
 x^2 - 2x + 6 \\
 x - 4 \overline{) x^3 - 6x^2 + 14x - 31} \\
 \underline{x^3 - 4x^2} \phantom{+ 14x - 31} \\
 -2x^2 + 14x - 31 \\
 \underline{-2x^2 + 8x} \phantom{- 31} \\
 6x - 31 \\
 \underline{6x - 24} \\
 -7
 \end{array}$$

$$\text{Quotient} = x^2 - 2x + 6$$

$$\text{Remainder} = -7$$

$$2. \quad P(x) = (x-4)^2(x)(x+3)$$

$$P(x) = a(x)(x+3)(x-4)^2$$

$$24 = a(5)(5+3)(5-4)^2$$

$$24 = 40a$$

$$a = \frac{24}{40} = \frac{3}{5}$$

$$P(x) = \frac{3}{5}x(x+3)(x-4)^2$$

$$3. \quad P(x) = 1(x-3)^2(x-5i)(x+5i)$$

$$= (x-3)^2(x^2+25)$$

$$= (x^2-6x+9)(x^2+25)$$

$$= x^4 - 6x^3 + 34x^2 - 150x + 225$$

$$4. \quad f(x) = \frac{7x-4}{x^2+x-14}$$

$$x^2+x-14=0$$

$$(x-3)(x+4)=0$$

$$x=3, x=-4$$

Vertical asymptotes  $x=3, x=-4$

Since the degree of the numerator is less than the degree of the denominator

Horizontal asymptote,  $y=0$

$$\frac{7x - 4}{x^2 + x - 12} = 0$$

$$7x - 4 = 0$$

$$7x = 4$$

$$x = \frac{4}{7}$$

$$x\text{-intercept} = (\frac{4}{7}, 0)$$

$$y = f(0) = \frac{7(0) - 4}{(0)^2 + 0 - 12} = \frac{1}{3}$$

$$y\text{-intercept} = (0, \frac{1}{3})$$

$$5. \quad \frac{5y}{(y-9)(y+3)} > 0$$

$$\frac{5y}{y^2 - 6y - 27} > 0$$

$$\frac{5y}{y^2 - 6y - 27} = 0$$

$$5y = 0$$

$$y = 0$$

$$(y-9)(y+3) = 0$$

$$y = 9, y = -3$$

| y        | $y < -3$ | $-3 < y < 0$ | $0 < y < 9$ | $y > 9$ |
|----------|----------|--------------|-------------|---------|
| function | -        | +            | -           | +       |

$$-3 < y < 0 \quad \text{or} \quad y > 9$$

$$\text{Interval} = (-3, 0) \cup (9, \infty)$$

$$6 \quad w^3 - 4w^2 - 16w < -64$$

$$w^3 - 4w^2 - 16w + 64 < 0$$

$$(w-4)^2(w+4) < 0$$

$$(w-4)^2(w+4) = 0$$

$$(w-4)^2 = 0 \quad \text{or} \quad w+4 = 0$$

$$w = 4$$

$$w = -4$$

| w        | $w < -4$ | $-4 < w < 4$ | $w > 4$ |
|----------|----------|--------------|---------|
| function | -        | +            | +       |

Solution:  $w < -4$

Interval:  $(-\infty, -4)$

$$7 \quad (a) \text{ Zeros: } x = -7, -3, 1, 4$$

$x = -7$  has 1

$x = -3$  has 2

$x = 1$  has 1

$x = 4$  has 2

(b) degree = 6

$$(c) \quad f(x) = a(x+7)(x+3)^2(x-1)(x-4)^2$$

$$7 = a(-6+7)(-6+3)^2(-8-1)(-6-4)^2$$

$$a = -\frac{1}{900}$$

$$= -\frac{1}{900} x^6$$

$$8 \quad g(x) = a(b)^x$$

$$g(0) = a(b)^0 = 5$$

$$a = 5$$

$$g(-2) = 5(b)^{-2} = \frac{5}{4}$$

$$b^{-2} = \frac{\frac{5}{4}}{5} = \frac{1}{4}$$

$$b^{-2} = \frac{1}{4}$$

$$b^2 = \frac{1}{\frac{1}{4}} = 4$$

$$b = \sqrt{4} = 2$$

$$g(x) = 5(2)^x$$

$$9. \quad 2 \log x - 4 \log (x^2 + 1) + 2 \log (x-1)$$

$$\log x^2 - \log (x^2 + 1)^4 + \log (x-1)^2$$

$$= \log \left( \frac{x^2 (x-1)^2}{(x^2 + 1)^4} \right)$$

$$10. \quad \ln \left( \frac{x^{35} y^{21}}{z^{21}} \right) = \ln x^{35} + \ln y^{21} - \ln z^{21}$$

$$= 35 \ln(x) + 21 \ln(y) - 21 \ln(z)$$